Patty Paper Assignment #1

NOTE TO THE STUDENT

Patty papers are thin, lightly waxed 5.5″ or 6″ squares of paper that you are going to use to make geometric discoveries. (Fast food restaurants use patty papers between the uncooked hamburger patties.) *Patty Paper Geometry* was designed for you to "learn by doing." You are going to be asked to read and follow the steps to a number of geometric investigations using patty papers. These investigations should lead you to discover most of the properties of geometric figures that are studied in high school geometry. For example: What do we know about isosceles triangles? What happens when a line intersects a pair of parallel lines? What is true about the diagonals of a rhombus?

Geometric properties are very important because we use geometry to help us understand the way things work. We can calculate perimeters, areas, and volumes only with the help of geometry. With geometry we can build bridges, skyscrapers, and airplanes. With geometry we can begin to understand why elephants have big ears or why grasshoppers can jump many times their height! With *Patty Paper Geometry* you will get a hands-on introduction to geometry.

You should keep a geometry notebook to organize all the investigations you do, the vocabulary you use, the conjectures you make, and the exercises and homework you complete. This will help you retain what you learn and will be your reference as you base new discoveries on what you've learned before.

What kinds of geometric properties can you discover given a supply of patty papers? Actually, any property that can be discovered using a compass and a straightedge can be discovered by folding patty papers. Tracing segments, angles, and polygons and using patty papers to compare lengths and angle measurements will become very useful methods of discovery for you. In *Patty Paper Geometry* you will discover most of the properties of high school geometry. What follows in *Patty Paper Geometry* are investigations leading to geometric discoveries followed by exercises in which you can apply your discoveries. Have fun!

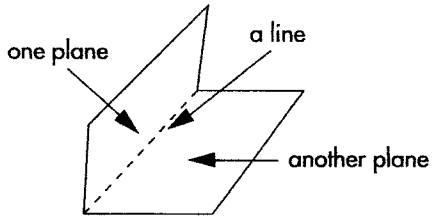
Michael Serra

BASIC PROPERTIES, DEFINITIONS, AND SYMBOLS

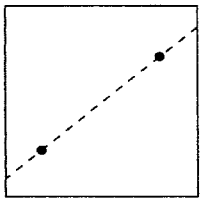
In *Patty Paper Geometry* you are going to discover geometric properties using patty papers instead of a compass and straightedge. First you will have to assume a few basic folding and congruence properties. Also, you need to agree on some basic definitions and become familiar with commonly used geometric symbols and notation. Carefully read this section and do the exercises before starting the investigations.

FOLDING PROPERTIES

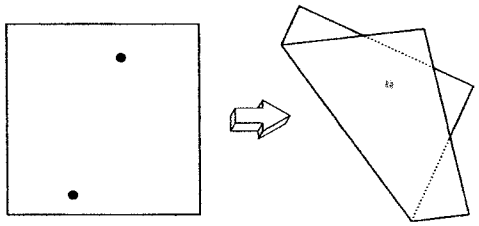
FP-1: It is possible to fold a patty paper so that the crease forms a line. This fold shows that the intersection of two planes is a line.



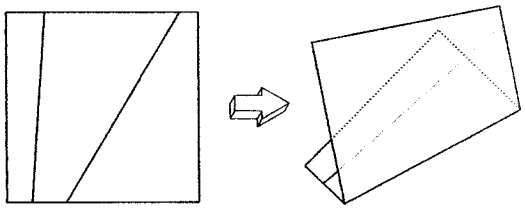
FP-2: If you draw two points on your patty paper, it is possible to fold the paper so that the crease (line) passes through the two given points. This property shows that two points determine a line.



FP-3: Patty paper can be folded so that a point on the paper can be placed over another point on the same paper.



FP-4: Patty paper can be folded so that a line (or a portion of the line) on the paper can be placed over another line on the same paper.

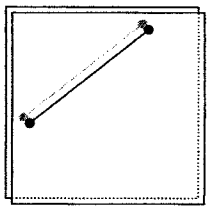


CONGRUENCE PROPERTIES

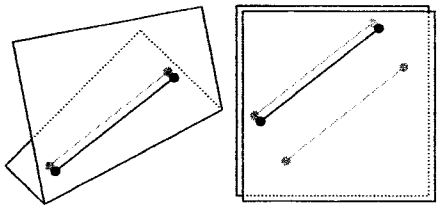
Definition: Two geometric figures are **congruent** if they have the same size and shape. If you place one of the figures on top of the other figure, then they will match exactly. The symbol for congruence is $\cong$.

With the ability to trace segments and angles from one patty paper to another, it is possible to duplicate segments, angles, and polygons. This also gives you the ability to check to see if two different segments are congruent or if two different angles are congruent. Here are the congruence properties of patty paper geometry.

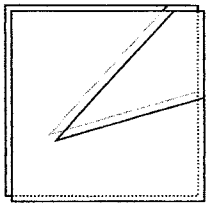
CP-1: A segment can be constructed congruent to another segment on a patty paper by tracing the original segment onto a second patty paper.



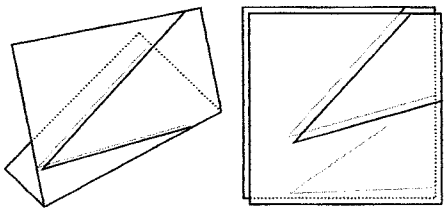
CP-2: Two segments are congruent if one segment can be placed exactly on top of the other by folding or if it can be traced onto another patty paper and the copy placed exactly on top of the other segment.



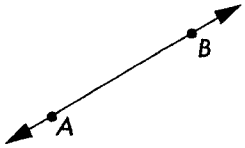
CP-3: An angle can be constructed congruent to another angle by placing a second patty paper over the angle and tracing the original angle onto the second patty paper.



CP-4: Two angles are congruent if one angle can be placed exactly on top of the other by folding or if it can be traced onto another patty paper and the copy placed exactly on top of the other angle.

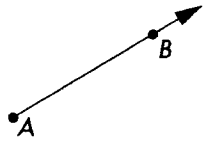


THE TERMS AND SYMBOLS USED IN PATTY PAPER GEOMETRY



Line AB

$\overleftrightarrow{AB}$

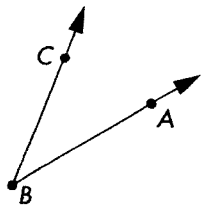


Ray AB

$\overrightarrow{AB}$

Segment AB

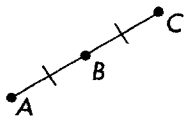
$\overline{AB}$



Angle ABC

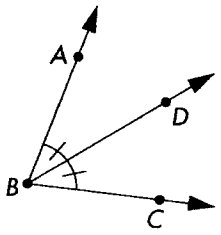
$\angle ABC$

When you compare the *sizes* (lengths or areas) of geometric figures, you will say the *measures are equal* or *not equal*. When you compare the *shapes* of geometric figures, you will say the *figures are congruent* or *not congruent*. When you label geometric figures, use similar slashes to mark segments that have the same length or angles that have the same degree measure.



The length of segment AB is equal to the length of segment BC ($AB = BC$).

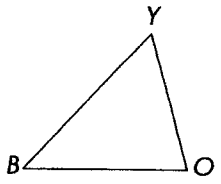
Line segment AB is congruent to line segment BC ($\overline{AB} \cong \overline{BC}$).

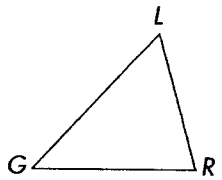


The measure of angle ABD is equal to the measure of angle DBC ($m\angle ABD = m\angle DBC$).

Angle ABD is congruent to angle DBC ($\angle ABD \cong \angle DBC$).

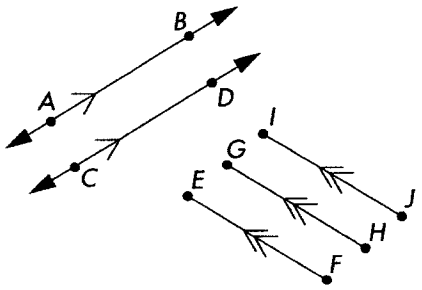
Two polygons are congruent if and only if they have all their corresponding angles congruent and all their corresponding sides congruent. The order of the letters in the statement of congruence tells you which segments and angles are corresponding and congruent.





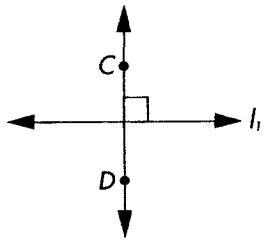
If $\Delta BOY \cong \Delta GRL$, then $\angle B \cong \angle G$, $\angle O \cong \angle R$, $\angle Y \cong \angle L$, $\overline{BO} \cong \overline{GR}$, $\overline{OY} \cong \overline{RL}$, and $\overline{BY} \cong \overline{GL}$.

When you wish to label lines or line segments as parallel, you mark the lines with similar arrow slashes. (The symbol for parallel is //.)



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ $\overleftrightarrow{EF} \parallel \overleftrightarrow{GH} \parallel \overleftrightarrow{IJ}$

When you wish to label lines as perpendicular, you mark the intersection with a small box to indicate the right angle. (The symbol for perpendicular is ⊥.)



$l_1 \perp \overleftrightarrow{CD}$

EXERCISE SET 0A

Your first task is to convince yourself that you can indeed perform the above construction properties.

1. Draw two points on a patty paper and fold so that the crease passes through the two points. (FP-2)
2. Draw two points on a patty paper and fold so that the two points lie on top of each other. (FP-3)
3. Draw two lines on a patty paper so that the lines go all the way to the ends of the patty paper and do not cross each other on the patty paper. Fold the patty paper so that the lines (or portions of the lines) lie on top of each other. (FP-4)
4. Draw two lines on a patty paper so that the lines go all the way to the ends of the patty paper and cross each other on the patty paper. Fold the patty paper so that the lines (or portions of the lines) lie on top of each other. (FP-4)
5. Draw a segment on a patty paper. Place a second patty paper over it and trace the original segment. (CP-1)
6. Draw an angle on a patty paper. Place a second patty paper over it and trace the original angle. (CP-3)

EXERCISE SET 0B

To review basic geometry vocabulary, match each term with the diagram that best illustrates it.

1. _____ point
2. _____ line
3. _____ plane
4. _____ ray
5. _____ segment
6. _____ midpoint
7. _____ triangle
8. _____ acute angle
9. _____ obtuse angle
10. _____ right angle
11. _____ parallel lines
12. _____ perpendicular lines
13. _____ compass
14. _____ straightedge
15. _____ ruler
16. _____ protractor